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Hence every single root c , of the equation $f(z) = 0$, is also a root of the equation $f'(z) - C = 0$, if the true value, viz.,

$$C = \frac{2\pi i}{\int_c \frac{dz}{f(z)}},$$

be substituted for C .

The fencing in, (*einhegung*), or determination of each single root c , and the numerical calculation of the marginal integrals around the areas in which they lie, need not be explained here, as their practical admissibility (*ausföhrbarkeit*) is theoretically unquestioned.

Nor need the train of consequences, which might be drawn from the foregoing, be enumerated here, the following only need be taken into consideration. (*platzfinden*.)

The processes (*mittel*) are known by which to reduce one *algebraic* equation to another, to wit, to $f(z) = 0$, which contains all the roots of the former as single roots; we also know how to determine (*einzuhegen*) each one of the latter.

Whenever this is done our formula in question presents the root $z = c$ as a root of the equation $f'(z) - C = 0$, which is lower than the former by one degree.

This equation may then be treated in like manner as the former, *and then we arrive, by a finite number of entirely definite and practical operations, at an equation of the first degree, viz.; $z = c$, in which the value of c is composed of the marginal integrals in a form similar to a continued fraction.*

Moreover we obtain $z - c$ as a common divisor of $f(z)$ and of $f'(z) - C$.

RECTIFICATION OF THE HYPERBOLA.

BY ARTEMAS MARTIN, M. A., ERIE, PA.

THE equation to the hyperbola, in rectangular coordinates referred to its center, is

$$b^2x^2 - a^2y^2 = a^2b^2. \quad (1)$$

Denoting by H_x the arc whose abscissa is x ,

$$\begin{aligned} dH_x &= \left(1 + \frac{dy^2}{dx^2}\right)^{\frac{1}{2}} dx = \left(\frac{[1 + (b^2 \div a^2)]x^2 - a^2}{x^2 - a^2}\right)^{\frac{1}{2}} dx, \\ &= \left(\frac{e^2x^2 - a^2}{x^2 - a^2}\right)^{\frac{1}{2}} dx = a \left(\frac{e^2v^2 - 1}{v^2 - 1}\right)^{\frac{1}{2}} dv, \end{aligned}$$

where e , the eccentricity, $= \frac{(a^2 + b^2)^{\frac{1}{2}}}{a}$, and $av = x$.

Put $e = \frac{1}{e}$ and $v = \left(\frac{1-e_1^2 v_1^2}{1-v_1^2}\right)^{\frac{1}{2}}$, then by substitution,

$$dH_x = \frac{a(1-e_1^2)dv_1}{(1-v_1^2)^{\frac{3}{2}}(1-e_1^2 v_1^2)^{\frac{1}{2}}}. \quad (2)$$

Let $A = v_1 \left(\frac{1-e_1^2 v_1^2}{1-v_1^2}\right)^{\frac{1}{2}}$, then

$$dH_x = adA - a \left(\frac{1-e_1^2 v_1^2}{1-v_1^2}\right)^{\frac{1}{2}} dv_1 + \frac{a(1-e_1^2)dv_1}{(1-v_1^2)^{\frac{1}{2}}(1-e_1^2 v_1^2)^{\frac{1}{2}}};$$

$$\therefore H_x = aA - aE(e_1, v_1) + \int \frac{a(1-e_1^2)dv_1}{(1-v_1^2)^{\frac{1}{2}}(1-e_1^2 v_1^2)^{\frac{1}{2}}}, \quad (3)$$

where $E(e_1, v_1)$ denotes an elliptic arc, semimajor axis unity, eccentricity e_1 and abscissa v_1 .

Let $e_1 = \frac{1-\sqrt{1-e_2^2}}{1+\sqrt{1-e_2^2}}$ and $v_1 = \frac{2v_2}{1-e_2^2} \left(\frac{1-v_2^2}{1-e_2^2 v_2^2}\right)^{\frac{1}{2}}$, and we get

$$\begin{aligned} \frac{a(1-e_1^2)dv_1}{(1-v_1^2)^{\frac{1}{2}}(1-e_1^2 v_1^2)^{\frac{1}{2}}} &= 2ae_1 dv_1 + 2a \left(\frac{1-e_1^2 v_1^2}{1-v_1^2}\right)^{\frac{1}{2}} dv_1 \\ &\quad - 2a(1+e_1) \left(\frac{1-e_2^2 v_2^2}{1-v_2^2}\right)^{\frac{1}{2}} dv_2. \end{aligned} \quad (4)$$

$$\therefore H_x = aA + 2ae_1 v_1 + aE(e_1, v_1) - 2a(1+e_1)E(e_2, v_2). \quad (5)$$

See *Wright's Solutions of the Cambridge Problems*, vol. ii, pp. 148—9.

A PROBLEM IN LEAST SQUARES.

BY R. J. ADCOCK, MONMOUTH, ILL.

FIND the most probable position of the straight line determined by the measured coordinates, each measure being equally good or of equal weight, (x_1, y_1) , (x_2, y_2) , ... (x_n, y_n) of n points, that is find a and b in the equation

$$y = ax + b. \quad (1)$$

Since $y_1 - ax_1 - b$, $y_2 - ax_2 - b$, ... $y_n - ax_n - b$ are the distances, parallel to the axis y , from the n points to the required line,

$$\left[(y_1 - ax_1 - b)^2 + (y_2 - ax_2 - b)^2 + \dots + (y_n - ax_n - b)^2 \right] \frac{1}{1-a^2} = S(d_1^2) = u, \quad (2)$$

is the sum of the squares of the normals from the n points to the required line, which sum by the principle of least squares, ANALYST, p. 183, Vol. IV, must be a minimum. Therefore